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B.M.S COLLEGE FOR WOMEN, AUTONOMOUS

BENGALURU – 560004

SEMESTER END EXAMINATION – SEPT/OCTO-2023

M.Sc. in Mathematics-2nd Semester

ALGEBRA –II

Course Code: MM201T **QP Code: 12001**

Duration: 3 Hours **Max marks: 70**

Instructions: ***1) All questions carry equal marks.***
2) Answer any five full questions.

1. (a)Define (i) Nil radical $N(A)$, (ii) Jacobson radical $J(A)$ of a A. Prove that $x \in J(A)$ if and only if $1 - xy$ is a unit in A for all $x \in A$.

(b) Define extension and contraction of ideals with respect to a ring. If C denote the set of all contracted ideals I of A and E denotes the set of all extended ideals J of B then show that $C = \{I : I^{ec} = I\}$ and $E = \{J : J^{ce} = J\}$.

(c) Explain the following (i) Operations on ideals. (ii) Prime spectrum of a ring A with suitable example for each.

(5+5+4)

2. (a) Show that the sub module of a unital R module M generated by a subset S of M consists of all linear combination of elements in S.

(b) State and prove fundamental theorem of Homomorphism on Modules.

(c) (i) Prove that the kernel of a homomorphism is a sub module.(ii) Prove that the range of a homomorphism is a sub module.

(5+5+4)

3. (a) State and prove Nakayama Lemma.

(b) Define a simple module. Show that an A-module M is simple if and only if $M \cong \frac{A}{I}$ as for some maximal ideal I of A .

(7+7)

4. (a) Prove that A-module M is of finite length if and only if it is both Noetherian and Artinian.

(b) Prove that a commutative ring with identity is Noetherian if and only strictly ascending chain of ideals is of finite length.

(7+7)

5. (a) Define an algebraic extension of a field. If L is an algebraic extension of K and K is an algebraic extension of F then prove that L is an algebraic extension of F.

(b) Prove that the elements in an extension K of a field F which are algebraic over F form a subfield of K.

(c) Let $a = \sqrt{2}, b = \sqrt[4]{2}$, where R is an extension of Q. Verify that $(a + b)$ and (ab) are algebraic of degree atmost $(deg\ a)(deg\ b)$.

(5+5+4)

6. (a) Show that it is impossible to construct a heptagon .

(b) Let $T: F \to F'$ be an isomorphism given by $\alpha T = \alpha',\ \forall \alpha \in F$. Then prove that $T^*: F[x] \to F'[t]$ is a continuation isomorphism of T.

(c) Determine the splitting field of $x^4 - 2$ over Q.

(5+5+4)

7. (a) Show that the polynomial $f(x) \in F[x]$ has multiple roots if and only if $f(x)$ and $f'(x)$ have a non-trivial common factors.

(b) Prove that any finite extension of a field of characteristic 0 is simple extension.

(c) Show that any field of characteristic 0 is perfect field.

(6+4+4)

8. (a) Define normal extension of a field. Prove that an extension K of a field F of degree two is normal.

(b) Define a fixed field. Let G be a sub group of group of automorphisms of a field K. Then show that fixed field of G is a sub field of K.

(c) If K is a finite extension of a field and if $G(K;F)$ is the group of all automorphisms of F then prove that $G(K;F)$ is finite and $O\big(G(K,F)\big) \leq [K:F]$.

(4+4+6)

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